VARDHAMAN MAHAVEER OPEN UNIVERSITY, KOTA

Rawatbhata Road, Kota (Rajasthan) - 324021

Post Graduate Degree Programme

M.A./M.Sc. Mathematics (Previous)

Internal Assignments

MA/MSc MT-01 To MA/MSc MT-05

Session : 2011 to 2012

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Previous)
Internal Assignments for MA/MSc MT-01 to MA/MSc MT-05

Dear Students,

The following internal assignments of various papers of M.A./M.Sc. Mathematics (Previous) are being send to you:

Programme Code	**Name of the Course/paper**
MA/MSc MT-01	Advanced Algebra
MA/MSc MT-02	Real Analysis and Topology
MA/MSc MT-03	Differential Equations, Calculus of Variations and Special Functions
MA/MSc MT-04	Differential Geometry and Tensors
MA/MSc MT-05	Mechanics

It is must to complete the internal assignments and after completion submit the assignments to the Director of your concerned Regional Centre either through your own presence or through registered speed post. Each internal assignment is of 20 marks, the marks obtains in internal assignment will be added with the marks obtained in term end examination. It is mandatory to complete the assignments in your own hand writing. There is no revaluation system for the internal assignment except technical mistakes. After submission of the assignment you will not be given the chance to improve the same or resubmit the same so try to give the best answer in your 1st attempt. Enclose the internal assignments of each paper/course in separate files and provide the below information on the first page of each file:

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Previous)

Scholar No. ☐☐☐☐☐☐☐☐☐☐☐

Name of Student ....Internal Assignment No. ....

Father's Name ....Programme Code ....

Address ....Name of the Course/Paper ....

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Name of Study Centre ....Assignment Submission Date..

Name of Regional Centre..

Note :

1. Use only A4 Size Paper for your response sheets.
2. Last date of Submission : Before one month from the date of commencement of Term-End Examinations.

Internal Assignment-I
MA/MSc MT–01
Advanced Algebra

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. If H, K are normal subgroups of G,

show that $\dfrac{G}{H \cap K}$ is isomorphic to a subgroup of $\dfrac{G}{H} \times \dfrac{G}{K}$

OR

Find the dual basis of the basis

$B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for $V_3(R)$ **(7 Marks)**

2. If G/H is abelian, then

show that $H \supset G^{(1)}$.

OR

Prove that in a Euclidean ring R, ideal generated by $\{a, b\}$, where $a(\neq 0)$, $b(\neq 0) \in R$, is a principal ideal generated by $c \in R$, where c is a greatest common divisor of a and b.

(7 Marks)

3. Attempt any **two** of the following . Each part consists of three marks.

(i) Write class-equation for the finite group G.

(ii) Define an inner product and an inner product space.

(iii) Define module.

(iv) Discuss Basic theory of field extensions. **(6 Marks)**

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Internal Assignment-II
MA/MSc MT–01
Advanced Algebra

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Let F be a field such that characteristic of F be $p > 0$. Also let $f(x) = x^p - \alpha$ be a polynomial in $F[x]$ with no root in F. Then prove that $f(x)$ is an inseparable polynomial.

OR

Let K be the splitting field of $f(x) = x^4 - 10x^2 + 1$ over Q. Then find $G(K \mid Q)$.

(7 Marks)

2. Let $V = R^3$ and $t : V \to V$ be a linear map, defined by $t(x, y, z) = (x + z, -2x + y, -x + 2y + z)$. What is the matrix of t with respect to basis $\{(1, 0, 1), (-1, 1, 1), (0, 1, 1)\}$?

OR

The matrix A of linear transformation $t : V \to V$ is diagonal if A is relative to a basis of V consisting of eigenvectors of t.

(7 Marks)

3. Attempt any two of the following:

(i) Discuss Cayley-Hamilton theorem.

(ii) Define an inner product and an inner product space.

(iii) What is Complete orthonormal set.

(iv) Prove that a orthogonal matrix is always non singular. **(6 Marks)**

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Internal Assignment-I
MA/MSc MT–02
Real Analysis and Topology

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. If $\mathcal{R}$ is a ring then show that the collection $\mathcal{F} = \left\{ A \in \mathcal{R} \ : \ A \text{ or } A^c \in \mathcal{R} \right\}$ is an algebra.

OR

If $< E_i >$ is a sequence of measurable sets with $m^* (E_i) = 0$; $i \in N$, then prove that $\bigcup_{i=1}^{\infty} E_i$ is a measurable set and has its measure zero. **(7 marks)**

2. If $< f_n >$ is a sequence of measurable functions, then show that $\lim \sup_n \langle f_n \rangle$ and $\lim \inf_n \langle f_n \rangle$ are also measurable.

OR

If E is the union of a countable family $\{E_i\}$ of pairwise disjoint measurable sets and if f is Lebesgue integrable over E, then prove that

$$\int_E f(x)\,dx = \sum_{i=1}^{\infty} \int_{E_i} f(x)\,dx.$$

(7 marks)

3. Attempt any two of the following . Each part consists of three marks.

(i) Show that the product of two square summable functions is summable.

(ii) State the Bessel's inequality.

(iii) Define L^p-space .

(iv) Define topological space .

(6 marks)

□ □ □

Internal Assignment-II
MA/MSc MT–02
Real Analysis and Topology

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Let $f : (X, \tau) \to (Y, \mu)$ be a map. Show that f is continuous if μ is an indiscrete topology.

OR

Let (X, τ) be a Hausdroff space and let $f : X \to X$ be continuous. Show that the set $\{x \in X : f(x) = x\}$ is closed in X. **(7 Marks)**

2. Show that every closed interval $[a, b]$ is compact with respect to relativised U-topology for $[a, b]$.

OR

Show that the one-point compactification of unit open interval (0, 1) is homeomorphic to the circle. **(7 Marks)**

3. Attempt any two of the following .Each part consists of three marks.

(i) Show that a cofinite topological space X is connected if X is infinite and disconnected if X is finite.

(ii) Show that the product of each family of locally compact space is locally compact.

(iii) Show that every filter base on a set X is contained in an ultrafilter on X.

(iv) Show that a subset A of a topological space X is open iff $A^\circ = A$. **(6 marks)**

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Internal Assignment-I
MA/MSc MT–03
Differential Equations, Calculus of Variations and Special Functions

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Prove that the extremal of $\int_a^b y\left(1+y'^2\right)^{1/2}$ is the catenary $y = a \cos h(ax + b)$

OR

Find the equation of the curve that passes through the point (3, 2, 1) and cut orthogonally the family of surfaces $x + yz = c$ **(7 Marks)**

2. Find the solution of Sturm-Liouville problem $y'' + \frac{1}{x} y' + \frac{\lambda}{x^2} y = 0, \quad 1 \le x \le 2$

with boundary conditions $y(1) = 0 = y(2)$

OR

Solve by the method of separation of variables :

$\frac{\partial u}{\partial x} = 4 \frac{\partial u}{\partial y}$; $u(0, y) = 8e^{-3y}$ **(7 Marks)**

3. Attempt any **two** parts

(a) Write general Laplace's equation.

(b) State the vandermonde's theorem.

(c) Define Legendre's function of first and second kind.

(d) Write Braf man's genarating function. **(6 Marks)**

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Internal Assignment-II
MA/MSc MT–03
Differential Equations, Calculus of Variations and Special Functions

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Find the extremal of the functional

$$I\left[x(t), y(t)\right] = \int_0^{\pi/4} \left(\dot{x}\dot{y} + 2x^2 + 2y^2\right) dt,$$

subject to the initial conditions at $t = 0, x = y = 0$; at $t = \frac{\pi}{4}$ $x = y = 1$.

OR

If the complete elliptic integral of the first kind is $K' = \int_0^{\pi/2} \left(1 - k^2 \sin^2 \phi\right)^{-1/2} d\phi,$

then show that $K' = \frac{\pi}{2}\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}; 1; k^2\right), \quad |k| < 1$ **(7 Marks)**

2. Prove that $F(a, b+1; c+1; x) - F(a, b; c; x) = \frac{a(c-b)x}{c(c+1)} F(a+1, b+1; c+2; x)$

OR

Prove that

$$\sum_{n=0}^{\infty} P_n(x) t^n = (1 - xt)^{-1}\, {}_1F_0\left[\frac{1}{2}; -; \frac{t^2\left(x^2 - 1\right)}{(1 - xt)^2}\right]$$

(7 Marks)

3. Attempt any **two** parts

(a) Define the Power Series Method

(b) Prove that: $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left\{\frac{\sin x}{x} - \cos x\right\}$

(c) Define the Hermite Differential Equation

(d) Write down the Associated Laguerre Differential Equation **(6 Marks)**

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Internal Assignment-I
MA/MSc MT–04
Differential Geometry and Tensors

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. For the curve $x = a(3t - t^3)$, $y = 3at^2$, $z = a(3t + t^3)$

Show that $\rho = \sigma = 3a(1 + t^2)^2$.

OR

From a point P on the conicoid $a^2x^2 + b^2y^2 + c^2z^2 = 1$ perpendiculars PL, PM, PN are drawn to the coordinate planes. Find the envelope of the plane LMN. **(7 Marks)**

2. Prove that the curves $du^2 - (u^2 + c^2)\, dv^2 = 0$ form an orthogonal system on the right helicoid $\vec{r} = (u \cos v, u \sin v, cv)$.

OR

Derive the differential equation of the asymptotic lines at a point (u, v) on the surface $\vec{r} = \vec{r}(u, v)$ in curvilinear coordinates. **(7 Marks)**

3. Attemt any two of the following :

(i) Write the equation of osculating plane in cartesian coordinates.

(ii) Define right conoid.

(iii) Explain curvilinear equation of a curve lying on a surface

(iv) Define lines of curvature. **(6 Marks)**

□ □ □

Internal Assignment-II
MA/MSc MT–04
Differential Geometry and Tensors

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Prove that on a surface with metric $ds^2 = a^2\, du^2 + b^2\, dv^2$ the geodesic curvature of the curve $u = c$ is $(ab)^{-1}\left(\frac{\partial b}{\partial u}\right)$.

OR

Prove that the permutation tensors are tensor of third order and also show that

$$\in_{ijk} = g_{il}\, g_{jm}\, g_{kn} \in^{lmn},$$

where the symbols have their usual meanings. **(7 Marks)**

2. Find out the differential equation of geodesic for the metric

$$ds^2 = f(x)\,dx^2 + dy^2 + dz^2 + \frac{1}{f(x)}dt^2 .$$

OR

Define Riemann's symbols of first and second kind. If B_i are components of a vector, prove that $B_{i,jk} - B_{i,kj} = B_\alpha R^\alpha_{ijk}$. **(7 Marks)**

3. Attemt any two of the following :

(i) Write Gauss's characteristic equation.

(ii) Define contravariant and covariant vectors.

(iii) Define intrinsic derivatives of a tensor.

(iv) Write the condition for asymptotic lines to be orthogonal. **(6 Marks)**

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Internal Assignment-I
MA/MSc MT–05
Mechanics

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. A rough uniform board, of mass m and length $2a$, rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other. Find the distance through which the board moves in this time.

OR

A rigid body can turn freely about an axis fixed in the body and in space. To find the moment of effective forces and kinetic energy about the axis of rotation.

$$\left[\text{Moment of effective forces } = M k^2 \ddot{\theta};\ K.E. = \frac{1}{2} M k^2 \dot{\theta}^2\right]$$

(7 Marks)

2. A cylinder rolls down a smooth plane whose inclination to the horizon is α, unwrapping as it goes a fine string fixed to the highest point of the plane, find its acceleration and the tension of the string.

OR

A uniform right ciruclar cone of vertical angle 2α moves under no force except at its vertex which is fixed. It is set rotating about a generater. Show that its axis describes in space a right cone of

angle 2β where

$$\tan\beta = \frac{1}{2}\tan\alpha + 2\cot\alpha$$

(7 Marks)

3. Attempt any two of the following :

(i) Define isntantaneous axis of rotation

(ii) State the Principle of Conservation of angular momentum under finite force.

(iii) Explain the Degree of Freedom

(iv) Mention the equation that is characteristic of harmonic motion (S.H.M.)

(6 Marks)

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Internal Assignment-II
MA/MSc MT–05
Mechanics

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. A particle of unit mass is projected so that its total energy is h, in a field of a force is a which the potential energy in $\phi(r)$ at a distance r from the origin. Find the differential equation of the path.

OR

The velocity components in a two dimensional flow field for an incompressible fluid are given by $u = -3y^2$ and $v = -6x$ then find the equation of stream line at the point (1, 1)

(7 Marks)

2. Show that the following velocity field is a possible case of irrotational flow of an incompressible fluid : $u = yzt$; $v = zxt$ and $w = xyt$

OR

Show that $\frac{x^2}{a^2} e^t + \frac{y^2}{b^2} \cos t + \frac{z^2}{c^2} e^{-t} \sec t = 1$ is a possible form for the boundary surface of a liquid. **(7 Marks)**

3. Attempt any two of the following :

(i) Define the conservative field of force.

(ii) Discuss relation between impulse and momentum.

(iii) Write the complex potential due to a doublet which makes an angle α with x-axis.

(iv) Define axis of spontaneous rotation. **(6 Marks)**

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