VARDHAMAN MAHAVEER OPEN UNIVERSITY, KOTA

Rawatbhata Road, Kota (Rajasthan) - 324021

Post Graduate Degree Programme

M.A./M.Sc. Mathematics (Previous)

Internal Assignments

MA/MSc MT-01 To MA/MSc MT-05

Session: 2010 - 2011

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Previous)
Internal Assignments for MA/MSc MT-01 to MA/MSc MT-05

Dear Students,

The following internal assignments of various papers of M.A./M.Sc. Mathematics (Previous) are being send to you:

Programme Code	**Name of the Course/paper**
MA/MSc MT-01	Advanced Algebra
MA/MSc MT-02	Real Analysis and Topology
MA/MSc MT-03	Differential Equations, Calculus of Variations and Special Functions
MA/MSc MT-04	Differential Geometry and Tensors
MA/MSc MT-05	Mechanics

It is must to complete the internal assignments and after completion submit the assignments to the Director of your concerned Regional Centre either through your own presence or through registered speed post. Each internal assignment is of 20 marks, the marks obtains in internal assignment will be added with the marks obtained in term end examination. It is mandatory to complete the assignments in your own hand writing. There is no revaluation system for the internal assignment except technical mistakes. After submission of the assignment you will not be given the chance to improve the same or resubmit the same so try to give the best answer in your 1st attempt. Enclose the internal assignments of each paper/course in separate files and provide the below information on the first page of each file:

Vardhaman Mahaveer Open University, Kota
Post Graduate Degree Programme
M.A./M.Sc. Mathematics (Previous)

Scholar No. ☐☐☐☐☐☐☐☐☐☐☐

Name of Student ....Internal Assignment No. ....

Father's Name ....Programme Code ....

Address ....Name of the Course/Paper ....

..

....

Name of Study Centre ....Assignment Submission Date..

Name of Regional Centre..

Note :

1. Use only A4 Size Paper for your response sheets.
2. Last date of Submission : Before one month from the date of commencement of Term-End Examinations.

Internal Assignment-I
MA/MSc MT–01
Advanced Algebra

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Let H and k be two subgroups of a group G such that k is normal in G. Then prove that

$$\frac{HK}{K} \cong \frac{H}{H \cap K}.$$

OR

Define composition series of a group and prove that there exists at least one composition series for every finite group G. **(7 Marks)**

2. Prove that every Euclidean ring is a principal ideal domain.

OR

Let V, V' and V'' be finite dimensional vector spaces over a field F and Let B, B' and B'' be their respective bases. Then for linear transformations $t : V \to V'$ and $s : V' \to V''$ prove that :

$$M_{B''}^{B}\left(sot\right) = M_{B''}^{B'}(s)M_{B'}^{B}(t).$$ **(7 Marks)**

3. Attempt any **two** parts of the following four parts. Each part consists of three marks.

(i) If M_1 and M_2 are submodules of an R-module M, then prove that $M_1 \cap M_2$ is a submodule of M.

(ii) Prove that the mapping $t : R^3 \to R^2$ given by

$$t\,(x, y, z) = (z, x + y) \;\forall\; (x, y, z) \in R^3$$

is a linear transformation.

(iii) Let K be a field extension of a field F and Let $\alpha \in K$ be algebraic over F. Then prove that any two minimal monic polynomial for α over F are equal.

(iv) Let V be an inner product space, and $S = \{v_1, v_2,\ldots, V_n\}$ be an orthonormal set in V. Then prove that for any vector $v \in V$, the vector

$$u = v - \sum_{i=1}^{n} v_i < v, v_i >$$

is orthogonal to each v_j, $j = 1, 2,\ldots, n$. **(6 Marks)**

□□□

Internal Assignment-II
MA/MSc MT–01
Advanced Algebra

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Let G_1 and G_2 be two groups and let N_1 and N_2 be normal subgroups of G_1 and G_2 respectively, then show that $N_1 \times N_2$ is a normal subgroup of $G_1 \times G_2$ and

$$\frac{G_1 \times G_2}{N_1 \times N_2} \cong \left(\frac{G_1}{N_1}\right) \times \left(\frac{G_2}{N_2}\right)$$

OR

Define solvable group and prove that every subgroup of a solvable group is solvable.

(7 Marks)

2. If a and b are any two non-zero elements of an Euclidean ring R, then shows that

(i) b is unit of $R \Rightarrow d(ab) = d(a)$

(ii) b is not unit of $R \Rightarrow d(ab) > d(a)$

where d is Euclidean mapping of R.

OR

Define rank and nullity of a linear transformation. Let V and V' be vector spaces over a field F. If $t : V \to V'$ be a linear transformation and if dimension of V is finite, then prove that

$$rank\,(t) + nullity\,(t) = dim\ V.$$ **(7 Marks)**

3. Attempt any two partes of the following four parts :

(i) Prove that every finite extension of a field is an algebraic extension.

(ii) Let $t : R^2 \to R^2$ be a linear transformation defined by

$$t\,(a, b) = (a + b, a - b).$$

Then find the matrix of t relative to basis

$$B = \{(1, 3), (1, 2)\} \text{ of } R^2.$$

(iii) Define eigenvalues and eigenvectors of a matrix. Let A be an $n \times n$ matrix over a field F. Then prove that a non-zero vector $X \in F^n$ is an eigenvector of matrix A if and only if there exists a scalar $l \in F$ such that

$$AX = X\lambda.$$

(iv) Let u and v be any two vectors in an inner product space V. Then prove that

$$\left|\, \| u \| - \| v \| \,\right| \leq \| u - v \|.$$ **(6 Marks)**

Internal Assignment-I
MA/MSc MT–02
Real Analysis and Topology

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Define a measurable set. If E_1 and E_2 are measurable sets, then prove that $E_1 \cup E_2$ and $E_1 \cap E_2$ are measurable sets.

OR

If f and g are two bounded and measurable functions defined on a measurable set E, then prove that :

$$\int_E \left(f(x) + g(x) \right) dx = \int_E f(x)dx + \int_E g(n)\, dx$$

(7 marks)

2. Let (X, τ) be a topological space and $A \subset X$. If A' is the set of all limit points of A, then show that $A \cup A'$ is a τ-closed subset of X.

OR

Define a normal space and prove that any homomorphic image of a normal space is a normal space. **(7 marks)**

3. Attempt any two partes of the following four parts :

Each part consists of three marks.

(i) If A be a set of all irrational numbers in closed interval [0, 3], then find measure of A, *i.e.* $m^*(A)$.

(ii) Define L_2-space and prove that if $f \in L_2$, $g \in L_2$, then $f + g \in L_2$.

(iii) Prove that every T_2-space is a T_1-space.

(iv) Define locally connected space and prove that every discrete space X is locally connected.

(6 marks)

□□□

Internal Assignment-II
MA/MSc MT–02
Real Analysis and Topology

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Prove that every open interval (a, b) is measurable.

OR

Let f be a summable function on the set E. Then prove that for each $\in > 0$, there exists a $\delta > 0$ such that

$$\left| \int_A f(x)\, dx \right| < \in$$

where A is a measurable subset of E with $m(A) < \delta$. **(7 Marks)**

2. Let (X, τ_1) and (Y, τ_2) be two topological spaces. Then prove that a function $f : X \to Y$ is continuous if and only if for every subset $A \subset X$,

$$f(\overline{A}) \subset \overline{f(A)}.$$

OR

Prove that every closed subset of a compact space is compact. **(7 Marks)**

3. Attempt any two partes of the following four parts :

Each part consists of three marks.

(i) For any two sets A and B, if $m^*(A) = 0$, then prove that $m^*(A \cup B) = m^*(B)$.

(ii) Define L_p-space and prove that if $f \in L_p$, $g \in L_p$, then $f + g \in L_p$.

(iii) Prove that a subset A of a topological space (X, τ) is open if and only if it is nbd. of each of its points.

(iv) Prove that every T_4-space is a T_3-space. **(6 marks)**

□□□

Internal Assignment-I

MA/MSc MT–03

Differential Equations, Calculus of Variations and Special Functions

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. *(a)* Solve $x^2 y \frac{d^2 y}{dx^2} + \left(x \frac{dy}{dx} - y \right)^2 - 3y^2 = 0$

(b) Solve the partial differential equation $y^2 r - 2ys + t = p + by$ by Morge's method.

OR

(a) Solve $(2x^3 - yz)\, dx + (2yz - xz)\, dy - (x^2 - xy + y^2)\, dz = 0$

(b) Discuss the nature of the partial differential equation. $yr + (x + y)\, s + xt = 0$ obtain its characteristic curves and reduce it to canonical for *m*. **(3+4 Marks)**

2. If $|z| < 1$, $Re\,(c) > Re\,(b) > 0$, then prove that

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}\, dt = B\,(b, c-b)\; {}_2F_1\,(a, b, c, z)$$

and hence deduce that

$${}_2F_1\,(a, b\,;\, c : z) = (1-z)^{-a}\; {}_2F_1\left(a, cob, c\,; \frac{z}{z-1} \right) \text{ for } |z| < 1 \text{ and } \left| \frac{z}{z-1} \right| < 1.$$

OR

(a) Prove that

$$\frac{d}{dx}\left[J_n^2(x) + J_{n+1}^2(x) \right] = 2\left[\frac{n}{x} J_n^2(x) - \frac{n+1}{x} J_{n+1}^2(x) \right]$$

(b) Prove that $\int_{-1}^{1} \left[P'n\,(x) \right]^2 dx = n\,(n+1)$ **(4 + 3 Marks)**

3. Attempt any **two** parts

(a) Prove that the eigen-functions of Sturm-Liouville system belonging to two different eigen-values are orthogonal with respect to weight function $r\,(n)$ in the given interval $[a, b]$.

(b) Write a short note on Isoperimetric problems.

(c) Write a recurrence relation for associated Laguerre polynomials and prove it.

(d) Find the values of $H'_{2n}(0)$ and $H'_{2n+1}(0)$. **(6 Marks)**

□□□

Internal Assignment-II
MA/MSc MT–03
Differential Equations, Calculus of Variations and Special Functions

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. ***(a)*** By reduction to a linear equation solve the Riccati's equation

$$\frac{dy}{dx} = -2 - 5y - 2y^2$$

(b) Solve the heat conduction problem

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

given that $u = 0$ when $t \to \infty$, $x = 0$ and $x = 1$.

OR

(a) Find the eigenvalues and eigenfunction of the sturm liouviue problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right] + \frac{\lambda}{x} y = 0 \text{ with } y(1) = 0,\ y(e^{\pi}) = 0.$$

(b) Find the extremals of the functional

$$I[y(x), z(x)] = \int_0^{\pi/2} [(y')^2 + (z')^2 + 2yz]\, dx$$

with the boundary conditions

$$y(0) = 0,\ y\left(\frac{\pi}{2}\right) = -1;\ z(0) = 0,\ z\left(\frac{\pi}{2}\right) = 1 \cdot$$

(3+4 Marks)

2. Solve the legendre's equation

$$(1 - x^2)\, y_2 - 2xy_1 + n\,(n + 1)\, y = 0$$

for large values of x, when n is a positive integer.

OR

(a) Prove the Beltrami's relation for legendre polynomials and deduce that

$$\int_{-1}^{1} \left(x^2 - 1\right) P_{n+1}(x) P_n'(n) = \frac{2n(n+1)}{(2n+1)(2n+3)}.$$

(b) State and prove the Curzen's integral. **(7 Marks)**

3. Attempt any **two** parts

(a) State the euler-lagrange's equation for an Extremal and use it to extremize the functional

$$I[y(x)] = \int_0^1 [(y')^2 + 12xy]\, dx$$

with $y(0) = 0, \quad y(1) = 1.$

(b) Explain the terms

(i) Eigenvalues,

(ii) Eigenfunctions and

(iii) Sturm-liouville boundary value problem.

(c) Show that $J_n(-x) = (-1)^n J_n(x)$

where n is a positive or negative integer.

(d) Prove that

$$e^{2xt-t^2} = \sum_{n=0}^{\infty} H_n(x) \frac{t^n}{\lfloor n}$$

valid for all finite x and t. **(6 Marks)**

□□□

Internal Assignment-I
MA/MSc MT–04
Differential Geometry and Tensors

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Prove that the indicatrix at a point of the surface $z = f(x, y)$ is a rectangular hyperbola if

$$(1 + p^2)\, t + (1 + q^2)\, r - 2pqs = 0.$$

OR

Find the length of the curve given as the intersection of the surfaces

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} 1, x = a \cos h\left(\frac{z}{a}\right).$$ **(7 Marks)**

2. Show that the edge of regression of the developable that passes through the parabolas $x = 0$, $z^2 = 4ay$, $x = a$, $y^2 = 4z$ is given by

$$\frac{3x}{y} = \frac{y}{z} = \frac{z}{3(a - x)}.$$

OR

Find the equation to the developable surface which has the following curves for their edge of regression

$$x = 6t,\ y = 3t^2,\ z = 2t^3.$$ **(7 Marks)**

3. Explain any two of the following :

(i) Developable surface,

(ii) Edge of regression,

(iii) First fundamental form,

(iv) Normal curvature. **(6 Marks)**

□□□

Internal Assignment-II
MA/MSc MT–04
Differential Geometry and Tensors

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. Prove that the Circle

$$lx + my+nz = 0,\ x^2+y^2+z^2 = 2cz$$

has three point contact at the origin will the paraboloid

$ax^2+by^2 = 2z,$ then $C= (l^2+m^2)/(bl^2+am^2)$

OR

For the curve $x = 3u,\ y = 3u^2,\ z = 2u^3$

Prove that $\rho = -\sigma = \frac{3}{2}(1+2u^2)$ **(7 Marks)**

2. Find the Inflexional tangent at (x, y, z) on the surface $y^2z = 4ax$

OR

Prove that curves of the family $\frac{v^3}{u^2}$ = Constant are geodesics on a surface with metric $v^2du^2 - 2uv\ du\ dv + 2u^2dv^2;\ (u>0,\ v>0)$ **(7 Marks)**

3. Explain any two of the following :

(i) Parrallel Surface

(ii) Kronecker Data

(iii) Metric Tensor

(iv) First Curvature **(6 Marks)**

□ □ □

Internal Assignment-I
MA/MSc MT–05
Mechanics

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. ***(a)*** Show that the centre of suspension and center of oscillation are interchangeable in case of compound pendulum.

(b) A body of mass M is acted upon by blow of Impulse I at a given point A. If u and v are the velocities of A in the direction of I just before and just after the action of I. Show that the change in kinetic Energy of the body is $\frac{1}{2}\, I\,(u+v)$.

OR

(a) A uniform rod is held in a vertical position with one end resting, and when released rotates about the end in contact with the table. Discuss the motion.

(b) An ellipse of axes a and b and a circle of radius bare cut from the same sheet of thin uniform metal and are superposed and fixed together with their centres coicident. The figure is free to move in its own vertical plane about one end of the major axis; show that the length of the equivalent simple pendulum is

$$\frac{(5a^2 - ab + 2b^2)}{4a}.$$

(4+3 Marks)

2. ***(a)*** Find the differential equation of a stream line in cartesian coordinates in the form $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$, where $u,\ v,\ w$ are the velocity components. Also find the condition such that orthogonal surfaces exist for these stream lines.

(b) Amass of fluid is in motion so that the lines of motion lie on the surface of coaxial cylinders. Show that the equation of continuity is

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho u)}{\partial\theta} + \frac{\partial(\rho v)}{\partial z} = 0,$$

where $u,\ v$ are the velocity components perpendicular and parallel to axis of z which is the axis of cylinder.

OR

(a) Define boundary surface. What is the condition for $f(x, y, z, r) = 0$ to be a boundary surface.

Show that $\frac{x^2}{a^2} f(t) + \frac{y^2}{b^2} g(t) + \frac{z^2}{c^2} h(t) = 1$ will be a boundary surface if

$$f(t) \cdot g(r) \cdot h(t) = 1.$$

(b) Define Euler's dynamical equations of motion for a fluid motion in cartesian coordinates.

(4+3 Marks)

3. Attempt any two of the following :

(i) Theorems of parallel axes for moment of inertia and product of inertia, Explain these.

(ii) Obtain an expression for the angular momentum of a particle of mass *m*.

(iii) Define stream lines and path lines. What is the difference between them.

(iv) Show that the motion specified by

$$u = \frac{3xz}{r^5}, v = \frac{3yz}{r^5}, w = \frac{3z^2 - r^2}{r^5} (r^2 = x^2 y^2 + z^2)$$

is a possible fluid motion for an in compressible fluid. **(6 Marks)**

□□□

Internal Assignment-II
MA/MSc MT–05
Mechanics

Max.Marks 20

Note: Attempt all the question. Question 1 & 2 carry 7 marks. Both questions have internal choice. Question number 3 has four short answer questions. Out of these you are required to answer any two questions. Question 3 carries 6 marks.

Attempt all three questions :

1. ***(a)*** Find the centre of percussion of triangle *ABC* which is free to move about its side *BC*.

(b) A uniform rod of length $2a$, is placed with one end in contact with a smooth horizontal table and is then allowed to fall, if α be the initial inclination to the vertical, show that its angular velocity when it is inclined at an angle θ is

$$\left\{\frac{bg}{a}\frac{\cos\alpha-\cos\theta}{1+3\sin^2\theta}\right\}^{1/2}.$$

OR

(a) A rigid body rotates about a fixed axis, find the moment of the effective forces about the axis of rotation.

(b) A uniform rod of mass misplaced at right angles to a smooth plane of inclination α with one end in contact with it. The rod is then released. Show that when the inclination plane is θ, the reaction of the plane will be

$$\left\{\frac{3(1-\sin\theta)^2+1}{(1+3\cos^2\theta)^2}\right\}mg\cos\alpha$$

(4+3 Marks)

2. ***(a)*** In the cartesian coordinates find the expressions for velocity of a fluid particle and the differential following the motion of the fluid.

(b) Obtain the equation of continuity for a fluid motion is cylindrical polar coordinates.

OR

(a) What do you mean by two dimensional motion ? Obtain an expression for stream function in two dimensions. What is the physical significance of a stream function.

(b) Amass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is

$$\frac{\partial p}{\partial t}+\frac{\partial(pw)}{\partial\theta}=0,$$

where w be the angular velocity of a particle whose azimuthal angle is θ at time t and ρ is the density of the fluid. **(4+3 Marks)**

3. Attempt any two of the following :

(i) Obtain an expression for the time of complete oscillation of a compound pendulum.

(ii) Discuss two dimensional motion of a rigid body under impulsive forces.

(iii) Define sources, singles and doubles. Also give expressions for their strengths.

(iv) Show that the ellipsoid.

$$\frac{x^2}{a^2k^2t^{2n}} + kt^n\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$$

is possible from of the boundary surface of a liquid. **(6 Marks)**

□□□