Program : M.A./M.Sc. (Mathematics)
M.A./M.Sc. (Previous)
Paper Code:MT-02
Real Analysis and Topology
Section – B
(Short Answers Questions)

1. Describe cantor set.

A. (P. 6)

2. Prove that every open interval is a Barel set.

A. (P. 16)

3. Prove that a σ-ring R of subset of a set X is a σ-algebra iff $X \in R$.

A. (P. 16)

4. Show that outer measure is translation invariant.

A. (P. 20)

5. If $\{E_n : n \in N\}$ is a sequence of disjoint measurable sets, then :

$$m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} m^*(E_i)$$

A. (P. 32)

6. Let $< E_i >$ be an infinite increasing sequence of measurable sets, then :

$$m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

A. (P. 39)

7. Giva an example to show that the function $|f|$ is measurable but f is not measurable.

A. (P. 50)

8. A function f defined on a measurable set E is measurable iff for any open set $G \subset R, f^{-1}(G)$ is a measurable et.

A. (P. 53)

9. If $< f_n >$ is a sequence of measurable functions defined on a measurable set E, then $sup_n < f_n > and\ inf_n < f_n >$ are also measurable on E.

A. (P. 59)

10. Let f be a measurable function defined on a set E. Then there exist a sequence $< g_n >$ of continuous functions on R such that $< g_n >$converges to f a.e. on E.

A. (P. 72)

11. The lower Lebesgue Darboux sums of any bounded measurable function f on a measurable set E can not exceed its upper Lebesgue Darboux sums.
A. (P. 83)
12. Show that every bounded measurable functions f defined on a measurable set E is L-integrable on E.
A. (P. 87)
13. If f is a bounded measurable function defined on a measurable set E, then $|f|$ is L-integrable over E and

$$\left|\int_E f(x)dx\right| \leq \int_E |f(x)|dx$$

A. (P.)
14. Let f be a bounded measurable function on a measurable set E and $f(x) \geq 0$ a.e. on E.
If $\int_E f(x)dx = 0$, then show that $f(x) = 0$ a.e. on E.
A. (P. 98)
15. Let $< f_n >$ be a sequence of measurable functions defined on a measurable set E, and $\lim_{n\to\infty} f_n(x) = f(x)$ a.e. on E. Then f is measurable on E.
A. (P. 105)
16. Let $< f_n >$ be a sequence of non-negative measurable functions. If $\lim_{n\to\infty} f_n(x_0) = f(x_0)$ at a point x_0 then for each $m \in N$

$$\lim_{n\to\infty} [f_n(x_0)]_m = [f(x_0)]_m$$

A. (P. 113)
17. Let f be a summable function on set E, then for given $\in > 0$, there exst a $\delta > 0$ such that $\left|\int_e |f(x)|dx\right| < \in$
Where e is a measurable subset of E with $m(e) < \delta$.
A. (P. 120)
18. Show that the space L_2 of a square summable function is a linear space.
A. (P. 125)
19. State and prove Minkowski's inequality in L_2 space.
A. (P. 127)
20. Let $< f_n >$ be a sequence in L_2. If $< f_n >$ converges in the mean square to a function $f \in L_2$, then $< f_n >$ converges in measurable to f.
A. (P. 128)
21. The scalar product in L_2 is a continuous function of its argument i.e. if $\{f_n\}$ and $\{g_n\}$ are two convergent sequences in L_2 with $\lim_{n\to\infty} f_n = f$ and $\lim_{n\to\infty} g_n = g$ then

$$\lim_{n\to\infty} < f_n, g_n > = < f, g >$$

A. (P. 135)
22. Let a set $D \subset L_2$ be everywhere dense in L_2. If Parseval's identity holds for all functions in D, then the system $\{\emptyset_i\}$ is closed.
A. (P. 142)

23. An orthonormal system $\{\emptyset_i\}$ is complete iff it is closed.
A. (P. 144)
24. Show that L^p-space is a linear space.
A. (P. 149)
25. Show that a sequence of functions in L^p-space has a unique limit.
A. (P. 157)
26. Let $< f_n >$ be a sequence of functions belonging to L^p-space. If this sequence is convergent, then it is a Cauchy sequence.
A. (P. 157)
27. Let X be a non void set. Let J be the family, consisting of $\emptyset$ and all those non-void subsets of X, whose complements are finite, then show that J is a topology for X which is known as cofinite topology.
A. (P. 162)
28. (Usual topology for R) Let R be he set of all real numbers. Let U be the family consisting of $\emptyset$ and all non-void subsets G of R having the property that for each $x \in G \exists$ an open interval $I_x\ s.t. x \in I_x \subset G$. then U is a topology for R.
A. (P. 162)
29. If A be a subset of a topological space (X, J) then $\bar{A} = A \cup A'$.
A. (P. 170)
30. Let $J = \{\emptyset, X, \{a\}, \{a,b\}, \{a,b,e\}, \{a,c,d, \{a,b,c,d\}\}$ be a topology on $X = \{a,b,c,d,e\}$, then
    (i) List all J-open subsets of X.
    (ii) List all J-closed subsets of X.
A. (P. 175)
31. In $\overline{A \cup B} = \bar{A} \cap \bar{B}$? Give reason in support of your answer.
A. (P. 177)
32. Let $J = \{\emptyset, X, \{1\}, \{1,2\}, \{1,2,5\}, \{1,2,3,4\}, \{1,3,4\}\}$ be the topology on $X = \{1,2,3,4,5\}$. Determine limit points closure, interior of the set $A = \{3,4,5\}$.
A. (P. 178)
33. Let B be a collection of subsets of a non-void set X. Then B is a base for some topology on X iff it satisfying the following conditions:-
    (i) $X = \cup \{B : B \in B\}$
    (ii) For any $B_1, B_2 \in B$ if $x \in B_1 \cap B_2\ then \exists B_3 \in B$ s.t. $x \in B_3 \subset B_1 \cap B_2$.
A. (P. 184)
34. A function $f : X \to Y$ is continuous iff the inverse image of every closed set of Y is a closed subset of X.
A. (P. 190)
35. A function $f : X \to Y$ is continuous iff for every subset $A \subset X$.

$$f(\bar{A}) \subset \overline{f(A)}$$

A. (P. 191)

36. Show that homeomorphism is an equivalence relation in the family of topological spaces.
A. (P. 193)
37. A one-one onto continuous map $f : (X, J) \rightarrow (\gamma, \xi)$ is a homeomorphism of f is either open or closed.
A. (P. 197)
38. Let $X = \{0, 1, 2\}, J = \{\emptyset, X, \{0\}\{0, 1\}\}$. Let f be continuous map of X into itself such that $f(1) = 0 \; and \; f(2) = 1, What \; is \; f(0)$?
A. (P. 197)
39. A topological space (X, J) is a T_1-space iff $\{x\}$ is closed, $\forall \; x \in X$.
A. (P. 201)
40. A finite subset of a T_1-space has no limit point.
A. (P. 203)
41. The property of a space being a Hausdroff space is a hereditary property.
A. (P. 208)
42. Show that every T_3-space is a T_2-space.
A. (P. 210)
43. Show that regularity is a topological property.
A. (P. 212)
44. Show that a closed sub space of normal space is a normal space.
A. (P. 214)
45. A closed subset of a compact space is compact.
A. (P. 220)
46. Show that a compact space has Bolzano-Weiers trass property.
A. (P. 227)
47. A compact Hausdorff space if normal.
A. (P. 228)
48. Show that every compact topological space is locally compact, but converse is not necessarily true.
A. (P. 229)
49. Every open continuous image of a locally compact space is locally compact.
A. (P. 229)
50. Let (X_∞, J_∞) be the one-point compactification of a topological space (X, J), then (X, J) is uniquely embedded into (X_∞, J_∞) such that $X_\infty \sim X$ is a singleton.
A. (P. 235)
51. Let (X_∞, J_∞) be the one-point compactification of a topological space (X, J) then X is a subspace of X_∞.
A. (P. 235)
52. The one point compactification of the plane is homeomorphic to the sphere. (P. 237)

53. Let (X, J) be a topological space and (γ, J_γ) be its subspace. Let A and B be two subsets of γ then A and B are J_γ-separated iff A and B are J-separated.
A. (P. 240)
54. Two closed subsets of a topological space are separated iff they are disjoint.
A. (P. 241)
55. A topological space X is disconnected iff A is the union of two non-void disjoint open (closed) sets.
A. (P. 244)
56. Let G be a connected subset of a topological space (X, J). H is a subset of X s.t. $G \subset H \subset \overline{G}$, then H is connected.
A. (P. 246)
57. Give an example of a locally connected space which is not connected.
A. (P. 251)
58. The image of a locally connected space under a open continuous mapping is locally connected.
A. (P. 252)
59. Let (X, J) and (γ, ξ) be two topological spaces and $(X \times \gamma, (P))$ be the product space of X and γ. Then the projection mappings $\pi_x \; and \, \pi_y$ are continuous and open mappings.
A. (P. 257)
60. The product space $(X \times \gamma, P)$ is Hausdorff if the space (X, J) and (γ, ξ) are Hausdorff.
A. (P. 259)
61. The product space $(X \times \gamma, P)$ is connected if X and γ are connected.
A. (P. 259)
62. Let X be a product space of an arbitrary collection $\{(X_\lambda, J_\lambda) : \lambda \in \Lambda\}$ of topological spaces. Then J s the topology for X iff J is the smallest topology for which the projections are continuous.
A. (P. 263)
63. Let F be a finitely short family of open sets of a topological space (X, J). then ∃ a maximal finitely short sub family M of J such that $F \subset M$.
A. (P. 267)
64. A subset A of γ is closed in the quotient topology J_f relative to $f : X \rightarrow Y$ iff $f^{-1}(A)$ is closed in X.
A. (P. 269)
65. Let (X, J) be a topological space such that X/R is Hausdorff quotient space, then R is a closed subset of the product space $X \times X$ relative to product topology P.
A. (P. 272)
66. Let (X, J) be a topological space and let $x \in X$. Let N_x be the collection of all nbds of x. the show that $N_x$is directed by the inclusion relation $\underline{C}$.
A. (P. 277)
67. Let γ be subset of topological space (X, J). The set γ is J-open iff no net in X- γ converges to a point in γ.

A. (P. 278)

68. Let $X_0 \in X$ and J is the collection of all those subsets of X which contains X_0. Then show that J is a filter on X.

A. (P. 284)

69. Let $X = \{1, 2, 3, 4\} and\ C = \{\{1, 2\}, \{1.3\}\}$, then find base and filter taking C as a sub-base.

A. (P. 287)

70. Show that every filter J on a non-void set X is contained in an ultrafilter on X.

A. (P. 288)

71. Show that every subnet of an ultranet if an ultranet.

A. (P. 291)

72. Let J be a filter on a non-void set X and Let A is a subset of X, the ∃ a filter J^* finer that J such that $A \in J^*\ if\ A \cap F \neq \emptyset \forall F \in \exists^*$.

A. (P. 291)